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The Primitive and Imprimitive Soluble Subgroups of GL(4,P^k)

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Abstract:

In this paper we will determined all of the primitive and imprimitive Soluble Subgroups of GL(4,p^k). It turns out that the number of types of the irreducible Soluble Subgroups in GL(4,p^k)are 10 types and are M_i,i=1,...,10. moreover we find these subgroups.

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1. Section 1: Introduction and Elementary Definitions.

in the early 1871 Jordan determined a table containing the number of conjugacy classes of maximal irreducible soluble subgroups of GL(n,p), for p^n < 106.([6]) Dickson (1901, Chapter 12, pp. 260-287) determined all subgroups of PSp (2,p^k) and in (1904) he determined all subgroups of PSp (4 ,3). Mitchell (1914) determined the maximal subgroups of PSp(4,p^k) for odd p. Liskovec (1973) classified the maximal Irreducible (p,q)-subgroups of GL (r^2,p), where q and or are primes and q is odd. Colon (1977) determined the non - abelian q - subgroups (q prime) of GL (q,p^k) and the non - abelion 2 - subgroups of Sp (2,p^k). Harada and yamaki (1979) determined the irreducible subgroup of GL (n,2) for n À 6. KondratÈev (1985, 1986, and 1987) determined the irreducible subgroups of GL(7,2), the insoluble irreducible subgroups of GL(8,2) and GL (9,2) and the insoluble primitive subgroups of GL(10,2). In the early 1960 Sims developed an algorithm, based on coset enumeration, which takes as input a group G given by a finite representation and positive integer n, and output a list containing a representative of each conjugacy class of subgroups of G whose index is at most n. A similar algorithm was developed independently by Schaps (1968). After Kovacs, Neubauer and Newman (unpublished notes) have proposed an algorithm which computing certain maximal subgroups of low index. Now in this paper we will determine the irreducible Soluble Subgroups of Gl(4,p^k). For this purpose, we mention some Definitions and elementary notions.

1.1 Definition: let G,N and H be groups and G has a normal subgroup N_0 isomorphic to N such that G/N_0 is isomorphic to H, then we write G=N λ H. If G has a subgroup isomorphic to H which intersects N_0 trivially then G is a semidirect product of N and H, we denote by G=N*H.

1.2 Definition : We say that a group G has a central decomposition (H_1,...,H_n) if

1 - each H_i is a normal subgroup of G.

2 - G = H_1 ... H_n.

3 - for each i and j, $H_i \cap H_j \leq Z(H_i) \cap Z(H_j)$

4 - for each i and j, $H_i \cap H_j = Z(H_i)$ or $Z(H_j)$

We also say that G is the central product of H_i by G = H_1Y ... YH_n.

1.3. Definition: The holomorph of group G, denote by HOL(G), is the semidirect product of G and its automorphism group.

2. Section 2 : Notations and Elementary Results.

In this Section we discuss some necessary results, which needed for later sections to Notations and elementary notions for use of their in after chapter.

2.1. Notation: We use sym(X) by means the symmetric group on the set X, and Sn to means that the symetric group on the set of the first n positive integers. If G and H are permutation groups, we denote the wreath product of G and H by G wr H, where G is a co-ordinate subgroup and H is the top group.

2.2. Theorem: (Huppert (1967, Theorem II. 3. 2. p. 159). Let P be a prime, n top a positive integer and V be the vector space of dimention n over the field of p elements. If G is a subgroup of GL(V), denote by V * G the permutation group of degree p^n which is the semidirect product of V (acting on itself by translation) and G (acting in natural way) considered as a subgroup of sym(V). Let S be a complete and irredundant set of conjugacy class representatives of the irreducible soluble subgroups of GL(V).

(a) If $G \in S$. Then V * G is a primitive soluble permutation group of degree p^n .

(b) If $G \in S$ and H is a subgroup of GL(V), that is conjugate to G, then V * H is conjugate in sym(V) to V * G.

(c) If $G \in S$ and H is a subgroup of Gl(V), that is not conjugate to G, then V * H is not conjugate to V * G.

(d) If P is a primitive soluble subgroup of sym(V), then there is a group G in S such that V * G is conjugate to P.

We always take F to be a finite field with p^k elements and n a positive integer.

2.3. Theorem (See [4], [5] & [6]) : There exists an irreducible cyclic subgroup of order m in GL(n,F) if and only if m divides p^{kn} -1 and m dos not devide p^{kd} -1, for any positive integer d<n.

2.4. theorem (See [4], [5] & [6]): If there exist irreducible cyclic subgroups of order m in GL(n,F) then they lie in a single conjugacy class.

2.5. Definition: In GL(n,F) the irreducible cyclic subgroups of order P^{kn} -1 are called the signer cycles.

2.6. Definition: An extra special q-group is a finite non abelian q-group whose center, derived group and Frattini subgroup coincide and have order **q**.

2.7. Theorem: (See [3])

(a) Let G be an extraspinal q - group of order q1+2L and exponent q or 4. The group of automorphism S of G which acts trivially on both Z(G) and G/Z(G) is equal to Inn(G). Let H be the normal subgroup of Aut(G) consisting of those elements that act trivially on Z(G).

Then H/Inn(G) is isomorphic to a subgroup of the symplectic group Sp (2l,q).

If q is odd, then H/Inn(G) is isomorphic to the full symplectic group sp (2l, q), If G is the central product of l copies of D8, then H/Inn(G) is isomorphic to the orthogonal group $O^+(2l,2)$. If G is the central product of (l.1) copies of D8 and one Q_8, then H/Inn(G) is isomorphic to the orthogonal group $O^-(2l, Z)$

(b) Let G be the central product of a cyclic group of order 4 and extra special 2-group. The group of automorphisms of G that act trivially on both Z(G) andGZ(G) is equal to Inn(G). If H is the normal subgroup of Aut(G) consisting the those elements that act trivially on Z(G), then H/Inn(G) is isomorphic to the symplectic group Sp (2l,2).

Note that the group $O^{+}(2l,2)$ is the group of all linear transformations that preserve the quadratic form: $f(x_1, \dots, x_{2l}) = x_1x_2 + \dots + x_{2l-1}\, x_{2l}$ and the group $O^{-}(2l,2)$ is the group of all linear transformations that preserve the quadratic form:

$f(x_1, \dots, x_{2l}) = x_1x_2 + \dots + x_{2l-1}\, x_{2l} + x^2_{2l-1} + x^2_{2l}$

2.8. Definition: Let G be an irreducible subgroup of GL(n, F), acting on the vector space V. We call G imprimitive if there exists a decomposition $V=V_1+ \dots +V_r (r>1)$ of V that is preserred under the action of G.

We call the set $\{V_1 \dots V_2\}$ a system of imprimitivity for G, and each member of this set is called a block of imprimivity for G. The minimum of the set of dimensions of the blocks of imprimitivity for G is called the minimal block size of G. If G is not imprimitive. We call G primitive.

2.9. Theorem E :(see suprunenko (1976, theorem 15. 4. P.109) [11])

let M be an Imprimitive Maximal soluble subgroup of GL(n,F), and let $\Gamma := \{V_1 \dots V_2\}$ be an undefinable system of imprimitivity for M. Let θ: M $\rightarrow$ Sym(Γ) be the homomorphism defined by: $\forall g \in M$, $V_i\,(g\theta):=V_{ig}$. Then $N_M(V_1)|_{V1}$ is a primitive maximal soluble subgroup of GL(V_1), Mθ is a transitive maximal soluble subgroup of sym(Γ), and M is linearly isomorhic to $N_M\,(V_1)|_{V1}$ Wr Mθ.

2.10. Remark: Consider the case when m = n in the above theorem, then V_1 is 1 -dimensional, and so $N_M\,(V_1)|_{V1}$ = GL(l,F), by hypothesis M is irreducible, and therefore we must have $p^k > 2$. In particular GL(n, 2) contains no imprimitive subgroups if n is prime.

2.11. Theorem (see [4]): Let m be a proper divisor of n, let P_m be a completed and irredundant set of conjugacy class representatives of the primitive maximal soluble subgroups of GL(m, F) and let $\tau_{\frac{n}{m}}$ be a complete and irredundant set of conjugacy class represtatives of the transitive maximal soluble subgroups of $S_{\frac{n}{m}}$. Define the set S_m of imprimitive soluble subgroups of GL(n,F) by

$S_m := \{P \text{ wr } T | P \in P_m, T \in \tau_{\frac{n}{m}}\}$.

However, if $p^k = 2$, then define the set S_1 to be empty. Let S be the union of the Sm as m runs through the proper divisors of n. Then those members of S that are maximal soluble from a complete and irredundant set of conjugacy class representatives of the imprimitive maximal soluble subgroups of Gl (n, F).

2.12. Theorem: (See [13], lemma 19. 1. P.129, Theorem 20. 9, P.145). Let A be a maximal abelian normal subgroup of M. Then the following statements hold:

(a) A is conjugate to a group of block diagonal matrices, where each blak is the same, and is m by m, where m is a divisor of n:

(b) The linear span E, of the powers of any one of the m by m diagonal blocks of A is an extension field of FI_m.

(c) The degree of this field extension is m.

(d) A is isomorphic to the multiplicative group of E: in particular, A is cyclic of order p^{km} -1.

(e) A is the unique maximal abelian normal subgroup of M.

2.13 : Notation: Define the map ρ: N_L (F)$\to$ GL(2l,q) by

$$\rho(g)=\begin{bmatrix} \alpha_{11} & \beta_{11} & \cdots & \alpha_{1l} & \beta_{1l} \\ \gamma_{11} & \delta_{11} & \cdots & \gamma_{1l} & \delta_{1l} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \alpha_{l1} & \beta_{l1} & \cdots & \alpha_{ll} & \beta_{ll} \\ \gamma_{l1} & \delta_{l1} & \cdots & \gamma_{ll} & \delta_{ll} \end{bmatrix}$$

where:
$$\begin{aligned} u_i^{\,g} &= u_1^{\alpha_{i1}} v_1^{\beta_{i1}} \cdots u_1^{\alpha_{il}} v_l^{\beta_{il}} z^{\lambda_i} \\ v_i^{\,g} &= u_1^{\gamma_{i1}} v_1^{\delta_{i1}} \cdots u_1^{\gamma_{il}} v_l^{\delta_{il}} z^{\mu_i} \\ z^g &= z^v \end{aligned}$$

2.14. Theorem (See to [4], [5] & [6]):

Let n = q^l m. Where l>0, and q is a prime divisor of p^{km}-1. If q = 2, then suppos in addition that $P^{km} \equiv 1$(mod4). Let z1 be our fixed generator of a singer cycle of GL(m, F),and let a_1 be our fixed element of order m in GL(m, F) such that $a_1^{-1} z_1 a_1 = z_1^{p^k}$. Let a, z be the n by n block diagonal matrices with a_1 and z_1 running down their diagonals, respectively. Define the matrices u_i and v_i as Notatin 2.13 . Let S be the subgroup of GL(2l,q) That is generated by Sp(2l,q) and the block diagonal matrix with the matrix $\begin{bmatrix} 1 & 0 \\ 0 & p^k \end{bmatrix}$ running down its diagonal. let D be a completely reducible (not necessarily maximal) soluble subgroup of S which does not fix any non - zero isotropic subspace of the natural module for Sp(2l,q).

Suppose D has generating set {d_1,...,d_r}. If d_i is the matrix

$$\begin{bmatrix} \alpha_{11} & \beta_{11} & \cdots & \alpha_{1l} & \beta_{1l} \\ \gamma_{11} & \delta_{11} & \cdots & \gamma_{1l} & \delta_{1l} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \alpha_{l1} & \beta_{l1} & \cdots & \alpha_{ll} & \beta_{ll} \\ \gamma_{l1} & \delta_{l1} & \cdots & \gamma_{ll} & \delta_{ll} \end{bmatrix}$$

Then g_i be any matrix of GL(n, F) statistying

$$\begin{aligned} u_j^{\,g_i} &= u_1^{\alpha_{j1}} v_1^{\beta_{j1}} \cdots u_l^{\alpha_{jl}} v_l^{\beta_{jl}} z^{\lambda_j} \\ v_j^{\,g_i} &= u_1^{\gamma_{j1}} v_1^{\delta_{j1}} \cdots u_l^{\gamma_{jl}} v_l^{\delta_{jl}} z^{\mu_j} \end{aligned}$$

for some (arbitrary) integer γ_j and μ_j. let P the subgroup of GL(n, F) defined by: P:=<$G_{<a>}$(v_1,...,v_l),g_1,...,g_r,u_1.v_1,...,u_l,v_l,z> then P is the complete inverse image of D under r. Furthermore, P is primitive and has a maximal abelian normal

subgroup of order P^{km}-1. Now let D_1 be a complete and irredundant set of S – conjugacy class representatives of the completely reducible maximal soluble subgroup of S which do not fix any non - zero isotropic subspace of the natural module for sp (2l,q). Let P1 be the set of groups P obtained by the above method, one for each D, where D runs through the members of D_1. No two members of P are conjugate in GL(n, F). If M is a primitive maximal soluble subgroup of GL(n, F) whose unique maximal abelian normal subgroup has order P^{km}-1, then M is conjugate to a member of P_1.

2.15. Theorem: Let n = 2^lm, and suppose that $p^{km} \equiv 3 (\mathrm{mod} 4)$. Let z be our fixed generator of a singer cycle of GL(m, F), and let a_1 be our fined element of order m in GL(m,F) such that $a_1^{-1} z_1 a_1 = z_1^{p^k}$. Let a and z be the n by n block diagonal matriees with a_1 and z_1 running down their diagonals, respectively For 1≤i≤l-1 define the matrices u_i and v_i as notation 2.13.

Define u_l^+ and v_l^+ by: $u_1^+ = \begin{bmatrix} 0 & -I_m \\ I_m & 0 \end{bmatrix} \otimes I_2^{(l-1)}$ and $v_1^+ = \begin{bmatrix} 0 & -I_m \\ I_m & 0 \end{bmatrix} \otimes I_2^{(l-1)}$ and definee. u_l^- and v_l^- by $u_1^- = \begin{bmatrix} 0 & -I_m \\ I_m & 0 \end{bmatrix} \otimes I_2^{(l-1)}$ and $v_1^- = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix} \otimes I_2^{(l-1)}$ Where α and β are two elements of FI_m such that $\alpha^2+\beta^2= -I_m$. Let D be a completely reducible (not necessarily maximal) soluble subgroup of $O^+(2l,2)$ or $O^-(2l,2)$ which does not fix any non-zero isotropic subspace of the natural module for the relevant orthogonal group. Suppose D has generating set $\{d_1,\ldots,d_r\}$. If d_i is the matrix

$$\begin{bmatrix} \alpha_{11} & \beta_{11} & \cdots & \alpha_{1l} & \beta_{1l} \\ \gamma_{11} & \delta_{11} & \cdots & \gamma_{1l} & \delta_{1l} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \alpha_{l1} & \beta_{l1} & \cdots & \alpha_{ll} & \beta_{ll} \\ \gamma_{l1} & \delta_{l1} & \cdots & \gamma_{ll} & \delta_{ll} \end{bmatrix}$$

Then let g_i be any matrix of GL(n,F) satisfying

$$u_j^{g_i} = u_1^{\alpha_{j1}} v_1^{\beta_{j1}} \cdots (u_l^*)^{\alpha_{jl}} (v_l^*)^{\beta_{jl}} z^{\lambda_j}$$
$$v_j^{g_i} = u_1^{\gamma_{j1}} v_1^{\delta_{j1}} \cdots (u_l^*)^{\gamma_{jl}} (v_l^*)^{\delta_{jl}} z^{\mu_j}$$

for some (arbitrary) integers λ_j and μ_j, and where the superscript * is replaced by + or - according as D belongs to $O^+(2l,2)$ or $O^-(2l,2)$, respectively, let P be the subgroup of GL(n,F) defined by P:=<$C_{<a>}$ $(v_1,\ldots,v_l^*)$, $g_1,\ldots,g_r,u_1,v_1,\ldots,u_1^*,v_l^*$,z> then P is the complete inverse image of D under r. Furthermore, P is primitive and has a maximal abelian normal subgroup of order P^{km}-1. now let D^+ be a complete and irredundant set of $O^+(2l,2)$-conjugacy class representatives of the completely reducible maximal soluble subgroups of $O^+(2l,2)$ which do not fix any non-zero isotropic subspace of the natural module for $O^+(2l,2)$.

Define D^- similarly, with $O^-(2l,2)$ in place of $O^+(2l,2)$, let P_1 be the set of groups P obtained by the above method, one for each D, where D runs through the members of D^+ and D^-. No two members of P_1 are conjugate in GL(n,F). If M is a primitive maximal soluble subgroup of GL(n,F) whose unique maximal a belian normal subgroup hos order P^{km}-1. Then M is conjugate to a member of P_1.

Proof: The proof of this theorem goes exactly the similar with the proof previous theorem and the reader can be referred to [4] & [5].

2.16. Definition: Any group constructed by the methods theorems 2.11, 2.14 and 2.15 will be called a JS-maximal (for Jordan-suprunenko) of GL(n,F). we will also use the terms JS-imprimitive and JS-primitive to denote imprimitive and primitive JS-maximal, respectively. Note that every JS-maximal is irreducible and soluble. but not necessarily maximal soluble. The smallest value of p^{kn} for which there are JS-maximal and are not maximal soluble is 9. (see [11])

2.17. Remark: For the imprimitive we use P wr T where P and T are as described in Theorem A and For the primitives, we use $(C_{p^{km}-1}$ Y E)N D where E is extraspinal of order q^{1+2l} and exponent q or 4, and D is as described in theorem B or C. Of course, there may be many (pairwise non - isomorphic) groups with than a normal subgroup isomorphic to $C_{p^{km}-1}$ Y E whose quotient is isomorphic to D. But we always mean the one obtained by the construction methods in this paper.

2.18. Definition: Let the JS - maximal S of GL(n,F) be $M_1,\ldots,M_m$ and let G be an irreducible soluble subgroup of GL(n, F). If n is prime and G is cyclic, then we define the guardian of G to be that JS - maximal which is the normalizer of a singer cycle. Otherwise the guardian of G is defined to be M_i, where i is the least positive integer such that G is GL(n,F) - conjugate to a subgroup of M.

Section 3.

In this Section by using previous theorems and methods of Section **1** and section **2**, We determine The irreducible Soluble Subgroups of $GL(4,p^k)$. For this we proof the following main theorem.

3.1. theorem: Let p be a prime number and k,n be positive integers and let F be the filed of p^k elements. Then the number of types of The irreducible Soluble Subgroups in the group $GL(4,p^k)$ is 10.

Proof: By definition 2.16 since every JS-maximal is irreducible and Soluble, therefore, we suffices to determine the JS-maximal soluble subgroups of the $GL(4,p^k)$. For this purpose, let F be the filed of p^k elements. Since both GL(1,F) and S_2 are Soluble, therefore by theorem (2.11) and remark (2.17) there is exactly one Js-imperative soluble subgroup of GL(2,F), namely, $M_1(2,p^k):=GL(1,p^k)$ wr S_2 , $p^k\neq 2$

Also since the unique maximal abelian normal subgroup of the group $GL(2,p^k)$ order $p^{2k}-1$ or p^k-1, then by theorems 2.14, 2.15 and remark 2.17 the JS-Primitive group of order $p^{2k}-1$ and p^k-1,as follows:

$M_2(2,p^k)$: = $C_{p^{2k}-1} * C_2$,

$M_3(2,p^k)$: = $(C_{p^k-1}$ Y $Q_8)$ N $O^-(2,2)$, $p^k \equiv 3 (mod 4)$

$M_4(2,p^k)$: = $(C_{p^k-1}$ Y $Q_8)$ N Sp(2,2), $p^k \equiv 1 (mod 4)$

Therefore by used from JS - maximal $GL(2,p^k)$, the JS-imprimitives of $GL(4,p^k)$ listed as follows.

$M_1(4,p^k)$: = $GL(1, p^k)$ wr S_4,

$M_2(4,p^k)$: = $M_2(2,p^k)$ wr S_2 ,

$M_3(4,p^k)$: = $M_3(2,p^k)$ wr S_2 , $p^k \equiv 3 (mod 4)$,

$M_4(4,p^k)$: = $M_4(2,p^k)$ wr S_2 , $p^k \equiv 1 (mod 4)$,

And also The JS - primitive of $Gl(4,p^k)$ are listed below:

$M_5(4,p^k)$: = $C_{p^{4k}-1}$ * C_4 ,

$M_6(4,p^k)$: = $M_2(2,p^k)$ * C_2, $p^k \neq 2$,

$M_7(4,p^k)$: = C_{p^k-1} Y D_8 Y Q_8)N $O^+(4,2)$, $p^k \equiv 3(mod4)$,

$M_8(4,p^k)$: = C_{p^k-1} Y D_8 Y Q_8)N HOL(C_5) , $p^k \equiv 3(mod4)$,

$M_9(4,p^k)$: = C_{p^k-1} Y D_8 Y Q_8)N $O^+(4,2)$, $p^k \equiv 1(mod4)$,

$M_{10}(4,p^k)$: = C_{p^k-1} Y D_8 Y Q_8)N HOL(C_5) , $p^k \equiv 1(mod4)$.

When p^k = 2, then there are exactly 2 - irreducible Soluble Subgroup, namely M2 and M_5.

When p^k = 3, then there are exactly 7 - irreducible Soluble Subgroup, namely M_1, M_2, M_3, M_5, M_6, M_7, and M_8.

When p^k = 4, then there are exactly 4 - irreducible Soluble Subgroup, namely M_1, M_2, M_5 and M_6 .

When p^k = 5, then there are exactly 7 - irreducible Soluble Subgroup, namely M_1, M_2, M_4, M_5, M_6 .

and M_{10}

When p^k = 6, then there are exactly 4 - irreducible Soluble Subgroup, namely M_1, M_2, M_5, M_6.

When p^k = 7, then there are exactly 7 - irreducible Soluble Subgroup, namely M_1, M_2, M_3, M_5, M_6, M_7 and M_8 .

When p^k = 8, then there are exactly 4 - irreducible Soluble Subgroup, namely M_1, M_2, M_5, M_6 .

When p^k = 9, then there are exactly 7 - irreducible Soluble Subgroup, namely M_1, M_2, M_4, M_5, M_6, M_9, M_{10}.

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